Chapter 2
Reasoning and Proof

Section 5
Proving Statements about Segments

GOAL 1: Properties of Congruent Segments

A true statement that follows as a result of other true statements is called a theorem. All theorems must be proved. You can prove a theorem using a two-column proof. A two-column proof has numbered statements and reasons that show the logical order of an argument

THEOREM

THEOREM 2.1 *Properties of Segment Congruence*

Segment congruence is reflexive, symmetric, and transitive. Here are some examples:

REFLEXIVE For any segment AB, $\overline{AB} \cong \overline{AB}$.

SYMMETRIC If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

TRANSITIVE If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.
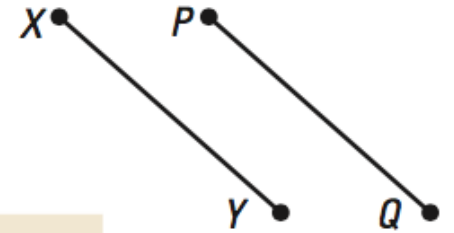
Example 1: Symmetric Property of Segment Congruence

   *need to address LENGTHS in addition to the congruence

You can prove the Symmetric Property of Segment Congruence as follows.

Given: $\overline{PQ} \cong \overline{XY}$

Prove: $\overline{XY} \cong \overline{PQ}$

Statements	Reasons
1. $\overline{PQ} \cong \overline{XY}$	1. Given
2. $PQ = XY$	2. Definition of congruent segments
3. $XY = PQ$	3. Symmetric property of equality
4. $\overline{XY} \cong \overline{PQ}$	4. Definition of congruent segments

*whenever you go from congruence to equality OR equality to congruence

   REASON → DEFINITION OF CONGRUENT _____segments/angles_____

A proof can be written in paragraph form, called paragraph proof. Here is a paragraph proof for the Symmetric Property of Segment Congruence.

Paragraph Proof You are given that $\overline{PQ} \cong \overline{XY}$. By the definition of congruent segments, PQ = XY. By the symmetric property of equality, XY = PQ. Therefore, by the definition of congruent segments, it follows that $\overline{XY} \cong \overline{PQ}$.

GOAL 2: Using Congruence of Segments

Example 2: Using Congruence
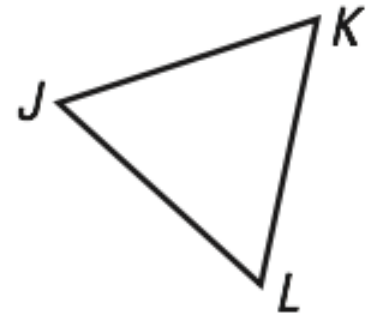
Use the diagram and the given information to complete the missing steps and reasons in the proof.

GIVEN ▶ $LK = 5$, $JK = 5$, $\overline{JK} \cong \overline{JL}$

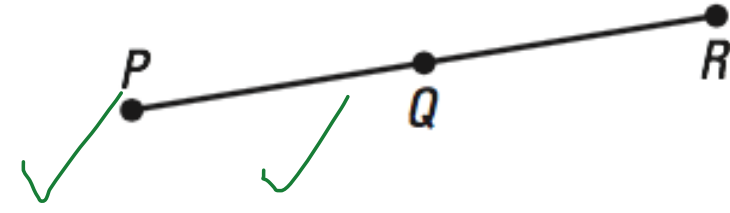**PROVE** ▶ $\overline{LK} \cong \overline{JL}$

Statements	Reasons
1. __a.__ $LK = 5$	1. Given
2. __b.__ $JK = 5$	2. Given
3. $LK = JK$	3. Transitive property of equality (substitution)
4. $\overline{LK} \cong \overline{JK}$	4. __c.__ Def. of ≅ Segments
5. $\overline{JK} \cong \overline{JL}$	5. Given
6. __d.__ $\overline{LK} \cong \overline{JL}$	6. Transitive Property of Congruence

Example 3: Using Segment Relationships

In the diagram, Q is the midpoint of PR. Show that PQ and QR are each equal to ½PR.

Given (handwritten annotation over "Q is the midpoint of PR")

PROVE (handwritten annotation over "PQ and QR are each")

Statements	Reasons
1. Q is the midpoint of $\overline{PR}$.	1. Given
2. $PQ = QR$	2. Definition of midpoint
3. $PQ + QR = PR$	3. Segment Addition Postulate
4. $PQ + PQ = PR$	4. Substitution property of equality
5. $2 \cdot PQ = PR$	5. Distributive property *(simplify; C.L.T.)*
6. $PQ = \frac{1}{2}PR$	6. Division property of equality
7. $QR = \frac{1}{2}PR$	7. Substitution property of equality